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THEORY AND CALCULATION FOR DISPLACEMENTS AND STRESSES AT AN EARTHQUAKE SOURCE

Paul G. Richards

Lamont-Doherty Geological Observatory

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

This progress report describes the method of calculation of ground motions near an earthquake source, and outlines the research which has led to ten-fold improvements over the previous computational methods. It is shown that surface waves are likely to contribute strongly to near-source ground-motions. A scalar problem is described, in terms of which one may better appreciate the essential difficulties of relating dislocation and traction on a rupturing fault surface.

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SUMMARY

During the first six months of the subject contract, the purposes of the research program were to investigate properties of an earthquake source, to provide a physical basis for discrimination between different types of seismic events. The two main approaches used were:

1. To compute theoretical seismograms which fit observed ground motion in the near-source region of an earthquake.

2. To investigate shear dislocations in the context of crack theory, requiring that associated shear stresses on the moving fault surface be dynamically satisfactory in terms of the mechanics of shear failure.

Research on the first of these approaches is heavily concerned with computational methods. The required theoretical seismograms involve, at each station, and at each time, a triple integration. Very considerable savings in computational effort can be made, if maximal efficiency is attained. By carefully arranging the sequence of integrations, and by using a variable number of points for each integration, we have achieved a tenfold improvement in the speed of these computations. We have also been able to extensively generalize the types of source motions for which resulting near-field ground motions may be computed. Present computations of this type are still limited by the assumption that the earthquake is taking place within an infinite elastic medium, rather than near the free surface of an elastic half space. Our present efforts are directed towards dropping this assumption, thus allowing the effect of the free surface to be computed. We have shown that this will be important, because we have found that elastic surface waves do contribute significantly to the strong ground-motion in the near field of an earthquake.

One of the least well known parameters of an earthquake is its stress drop. Theoretical considerations indicate that this quantity is proportional to the particle velocity at the source, and the constant of proportionality has been given in the literature as a complicated formula involving the rupture velocity for the earthquake, as well as the elastic parameters of the surrounding medium. We have succeeded in evaluating this relationship for a variety of rupture velocities.

The guiding principle behind our second major research approach, the theoretical study of shear dislocations which are compatable with the mechanics of shear failure, is as follows. The knowledge of the physics of failure on a fault surface gives us insight into the state of stress, and its time history during rupture, in the source region. However, to calculate local ground motions, we must first infer the dislocations on the fault surface which arise from the state of stress. This deduction, of the dislocations from the stress, is a mathematical problem in the field of singular integral equations. It is a subject about which, for three dimensional problems, little is known. Although the actual elasticity problem we here seek to solve is one which involves vectors, we have fortunately found an analog scalar problem which well illustrates the essence of difficulties in the theory. Our main research effort here has been directed towards finding what sub-region of the fault surface is controlling, via its stress history, the dislocation at each point on the fault. Although our main results to date are analytic, there is every indication that the problem will be amenable to a joint computational-analytic approach. Certainly, it appears that computers of the present generation are nowhere near big enough, for this problem to be solved by numerical studies alone.

Reports on Computational Procedures, for Generating Theoretical Seismograms from a given Fault Surface on which Dislocation is Already Specified as a Function of Position and Time

In this section, we present the improvements in recent programing. This work stems from the elastodynamic representation theorem in the form given by de Hoop (1958). A form of this theorem appropriate for the representation of a faulting source in an infinite homogeneous medium is discussed in detail by Haskell (1969). The source is described in terms of a shear dislocation propagating over the fault surface. This dislocation can be either an edge dislocaton, or a screw dislocation, and either of these two types will of course generate a three-component vector displacement at every point within the medium. Thus, six possible scalar equations must be considered, to include all possible displacements from either of the possible dislocations. An example of these formulae would be the following:

$$\begin{aligned} u_1^{(1)}(\underset{\sim}{x},t) = (\beta^2/4\pi) \iint_S \gamma_3 \Big\{ & 6(5\gamma_1{}^2 - 1)r^{-4}\int_{r/\alpha}^{r/\beta} D_1(\xi_1,\xi_2,t - t')t'dt' \\ & + 2(6\gamma_1{}^2 - 1)\ (\alpha r)^{-2}D_1(\xi_1,\xi_2,t - r/\alpha) \\ & - 3(4\gamma_1{}^2 - 1)\ (\beta r)^{-2}D_1(\xi_1,\xi_2,t - r/\beta) \\ & + 2\gamma_1{}^2\ (\alpha^3 r)^{-1}\dot{D}_1(\xi_1,\xi_2,t - r/\alpha) \\ & - (2\gamma_1{}^2 - 1)\ (\beta^3 r)^{-1}\dot{D}_1(\xi_1,\xi_2,t - r/\beta)\Big\} d\xi_1 d\xi_2 . \end{aligned} \qquad (1)$$

In this equation,

S = Fault plane area.

$\underset{\sim}{u} = (u_1, u_2, u_3)$ = Cartesian components of displacement measured from the initial state.

$\underset{\sim}{x} = (x_1, x_2, x_3)$ = Cartesian coordinates of point at which $\underset{\sim}{u}$ is to be evaluated.

$\underset{\sim}{\xi} = (\xi_1, \xi_2, \xi_3)$ = Cartesian coordinates of point of integration on S.

ρ = density.

α = P-wave velocity.

β = S-wave velocity.

$\underset{\sim}{n} = (n_1, n_2, n_3)$ = unit normal on S.

$\underset{\sim}{D} = (D_1, D_2, D_3)$ = displacement discontinuity across S.

The superscript $^{(1)}$, on the displacement calculated by equation (1), indicates that the assumed dislocation on S is an edge dislocation, for which $\underset{\sim}{D} = (D_1, 0, 0)$ only.

Equation (1), together with the five other similar equations for the other displacements components (three with the screw dislocation), has been very widely used for computations. It has almost always been assumed that $\underset{\sim}{D}$ has the form of a ramp function, the dislocation at any point of the fault surface thus growing steadily with time (after the rupture front has arrived), until the final fault offset is reached, at which time the fault is assumed to lock. With this special assumption for the time history of the dislocation, the basic computing formulae may be further reduced from the form (1), becoming even more amenable to programming. However, equation (1) does contain the generality permiting use of possibly more realistic dislocations. It will be noted that two of the terms in this basic equation contains the time derivative of the dislocation. Differentiation is an operation which, when done numerically, is liable to be performed inaccurately. Indeed, for some of the dislocations considered below, the time derivative actually contains singularities (which, however, are integrable). The numerical work requires that the resulting derivative be integrated over the

fault area, and this can be regarded as a smoothing process. However, we have greatly improved the stability of our computations by reversing the order of the differentiation and integration. A relevant term in equation (1) is thus treated as

$$\text{constant times } \frac{\partial}{\partial t}\iint_S 2\gamma_1^2 (\alpha^3 r)^{-1} D_1 (\xi_1, \xi_2, t - r/\alpha) d\xi_1 d\xi_2 \qquad (2)$$

The reversal of operations, as set out in equation 2, has yielded fourfold improvement over conventional programming of equation 1. That is, only one fourth the number of points on S are needed, to give answers with similar accuracy. More importantly, this order of numerical procedures (integration first, then differentiation) does permit the use of dislocations with more realistic time histories, and for which there may numerically be a singularity in the particle velocity of the fault surface.

In the conventional programming of equation 1 (see, for example, Anderson; 1974), a fixed number of points are taken in the ξ_1 and ξ_2 directions on the fault surfaces. For long, thin faults, perhaps as few as four points are taken in the ξ_2 direction. Since the basic computations (involving a total of 3 integrals: see equation 1) have to be performed for each moment in time, it would clearly be advantageous to allow a different spacing to be used for the points on the fault, for the calculation at each moment of time at the receiving position. To accomplish this, we now use Romberg integration for each of the three integrals. (Romberg integration uses successive doubling of the number of intervals used in a given integration problem. The result for each choice of intervals is stored. As the number of intervals is increased, a sequence of approximations to the desired

integration is found. Of course, the desired result would be obtained in the limit as interval size tends to 0. From the sequence of approximations to a desired integral, the algorithm predicts the limit to which the approximations are tending.)

The improvements resulting from use of triple Romberg integration, together with the sequence of operations stated in equation 2, have resulted in a tenfold improvement in computation time for a given problem (at a stated level of accuracy). Such a saving is highly significant, since, once a program of this type has successfully been debugged, it may well be used for problems requiring major amounts of computing time.

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A severe check on the accuracy of our present program is afforded by comparison with a particular problem already described by Richards (1973a, b). These papers describe a special method for finding theoretical seismograms which result from a growing plane elliptical shear crack. Our new, and much more general, method for doing these computations does successfully duplicate the wave shapes previously obtained by the specialized procedure.

The main weakness of our present computational method is its underlying assumption that the faulting occurs in an infinite homogeneous elastic medium. This rules out any effects due to the Earth's free surface. For example, surface waves are excluded, and P-SV coupling (in body waves) is also excluded. In fact, the energy radiated from a rupturing fault may well couple efficiently into surface waves. We justify this speculation as follows:

Consider the situation depicted in Figure 1. This shows parameters for a propagating source within an elastic medium, and generating surface waves. The displacement response at the receiver may be regarded simply as a superposition of all frequency components, and all positions occupied by the propagating source. Thus,

$$u(x, t) = \iint A(\xi, x, \cos\theta, \omega)e^{i\Phi}\, d\xi d\omega \tag{3}$$

in which the phase factor Φ is

$$\Phi = \omega\left(t - \int_0^{\xi} \frac{d\eta}{V(\eta)}\right) - kX(\xi, x). \tag{4}$$

Note that these equations permit a rupture velocity V which may vary along the path of faulting. In evaluating the double integral of equation 3, it may be expected that the major contribution will come from the point of stationary phase, obtained for each time t by equating partial derivatives of Φ (with respect to ξ and ω) to 0. Thus, the major contribution is from values of ω and ξ such that

$$t - \int_0^{\xi} \frac{d\eta}{V(\eta)} = \frac{X}{U} \tag{5}$$

and $$\frac{\omega}{k} = V(\xi)\, F\, (\xi, x, \cos\theta) \tag{6}$$

where $$F \equiv - \frac{\partial X}{\partial \xi} = \left(1 + \frac{x^2\sin^2\theta}{(x\cos\theta-\xi)^2}\right)^{-\frac{1}{2}}, \tag{7}$$

and U is group velocity.

The function F has value approximately equal to 1 if the angle θ is small. For distant receivers, F tends to the value $\cos\theta$.

Equation 5 states that surface waves arrive at a time given by the time to rupture to position ξ, plus the time taken to travel distance X with group velocity U. Equation 6 states that the frequency of such surface waves is that for which the phase velocity equals rupture velocity times the function F, which has a value between 0 and 1. These results can be summarized by saying that coupling into surface waves will be efficient if rupture velocity is somewhat greater than the surface wave phase velocity. Present estimates of the rupture velocity of faulting, about 2 or 3 kilometers per second, are indeed about the same as values for surface wave phase velocities.

Basic Theory for Finding Dislocations on a Fault Surface which are Consistent with a given Time History of Stress

The most commonly used dislocation in earthquake source theory probably is that due to Haskell (1969), the "ramp function" described in the section above. However, this time history probably would result in unacceptable stress singularities on the moving fault surface itself. A dislocation function is known, for which the associated stresses are constant on the moving part of the fault, and this function has been described by Kostrov (1964), and Richards (1973a, b). However, this dislocation is relevant only to the beginning rupture motions, and does not describe the way in which fault motions cease. As described in the original proposal for the present contract, one of the least understood aspects of earthquake source theory is the way in which fault motions cease. A fully numerical approach to this research problem is unlikely to be successful, since three spatial dimensions and one time dimension must be studied, and this total of four dimensions imposes enormous memory requirements for retaining the grid on which finite elements or finite difference computations are based. Frazier, in a paper presented at the 1974 Annual Meeting of the American Geophysical Union, indicates that even the computer ILLIAC 4 is inadequate. Thus, combined numerical and analytic methods provide the best chance for a successful approach.

Our analytic research, into the relationship between stress in an earthquake source region, and the resulting dislocation across the fault surface, has been advanced by finding an analog problem of relative simplicity. This analog problem involves a scalar field, rather than the vector displacement field of elasticity. The role of traction is played by a directional derivative of the scalar. We turn next to a basic exposition of this analog problem:

Suppose that $\phi = \phi(\underset{\sim}{x}, t)$ is some scalar function of space and time which satisfies the basic wave equation

$$\nabla^2\phi = \frac{1}{v^2} \frac{\partial^2\phi}{\partial t^2} \tag{8}$$

within a region V, and $\phi_o(\underset{\sim}{x})$ is the initial $(t \leq 0)$ static value of ϕ. At time $t = 0$, ϕ begins to experience a growing dislocation across an internal surface Σ, which may be regarded as the fault surface, and is contained in V. However, although ϕ is discontinuous across Σ, $\frac{\partial\phi}{\partial n}$ is kept continuous (in anaology with the requirement of continuity of traction across a fault surface). The problem is, first, to find the resulting ϕ $(\underset{\sim}{x}, t)$ throughout V in terms of the discontinuity, in ϕ on Σ. The later problem will be, to specify the value of $\frac{\partial\phi}{\partial n}$ on Σ, and to deduce an associated discontinuity in ϕ. (This second problem is the analogue of specifying the history of traction on a rupturing fault surface, and finding the related discontinuity in displacement.)

An important role in these problems is of course played by Green's function $G(\underset{\sim}{x}, t; \underset{\sim}{\xi}, \tau)$, which in V satisfies

$$\nabla^2 G = \frac{1}{v^2} \frac{\partial^2 G}{\partial t^2} + \delta(\underset{\sim}{x}-\underset{\sim}{\xi})\delta(t-\tau), \tag{9}$$

the δ's being Dirac delta functions. The "infinite space" Green's function satisfying (9) is

$$G = -\frac{1}{4\pi R} \delta\left(t-\tau-\frac{R}{v}\right) \tag{10}$$

where R is the distance $|x-\xi|$.

We have found the solution to the first problem in three different forms, each of which has merit. They are

$$\phi\,(x,t) = \iint_{\Sigma} \frac{\cos\theta}{4\pi R}\left\{\frac{1}{v}\left[\dot{\phi}(\underset{\sim}{\xi},\, t-\frac{R}{v})\right] + \frac{1}{R}\left[\phi(\underset{\sim}{\xi},\, t-\frac{R}{v})\right]\right\} d\Sigma$$

$$= -\frac{1}{4\pi}\iint_{\Sigma} \cos\theta\ \frac{\partial}{\partial R}\left\{\frac{1}{R}\left[\phi(\underset{\sim}{\xi},\, t-\frac{R}{v})\right]\right\} d\Sigma$$

$$= \frac{1}{4\pi}\iint_{\Sigma} \frac{\partial}{\partial n}\left\{\frac{1}{R}\left[\phi(\underset{\sim}{\xi},\, t-\frac{R}{v})\right]\right\} d\Sigma \qquad (11)$$

in which [ ] denote the discontinuity taken across Σ in the direction of $\underset{\sim}{n}$, the latter being the unit normal, and θ, R, $\underset{\sim}{x}$, $\underset{\sim}{\xi}$ are as shown in Figure 2.

Virtually all the practical problems contained in the elasticity problem are present also in the uses of equations (11), but these analogue forms do not have the algebraic complexity of the vector problems. A list of these problems, now being actively pursued, include the following:

(i) To study the singularities in (11), as the receiver position $\underset{\sim}{x}$ is taken ever closer to the fault surface Σ. The problem here is that the quantity R tends to 0 for some part of the surface integration. Since the integrand contains terms like R^{-2}, this implies non-integrable singularities. In this connection, it is important to note that if the term in square brackets, in the last of equations 11, is a constant, then the resulting integral is nothing but the solid angle subtended by Σ at $\underset{\sim}{x}$. This, of course, is simply 2π, if $\underset{\sim}{x}$ is on the fault surface.

(ii) To study the region of fault surface actually contributing to the integrals at a specified time. The point ~~here~~ is that all the integrands in equations 11 (and in Haskell's equation (1) above), are evaluated for retarded time. This

retarded time, if R is large enough, can then be so early, that for the point $\underset{\sim}{\xi}$ of integration, the rupture front has not yet arrived, and the dislocation is zero. Thus, a large part of the fault surface may not actually be contributing to the integration. The parts of Σ which contribute to the integration are defined by $\underset{\sim}{\xi}$ values satisfying the inequality

$$t - \frac{R}{v} \geq \tau(\underset{\sim}{\xi}) \tag{12}$$

where τ is the time at which the rupture arrives at $\underset{\sim}{\xi}$. We have examined the case of earthquake motions initiated on the fault surface Σ by a dislocation which begins at a point, and grows behind a circular rupture front, which expands with rupture velocity $c<v$. The sub-region of Σ which influences traction at a particular point on Σ can then be found analytically, by solving equation (12) for (ξ_1, ξ_2).

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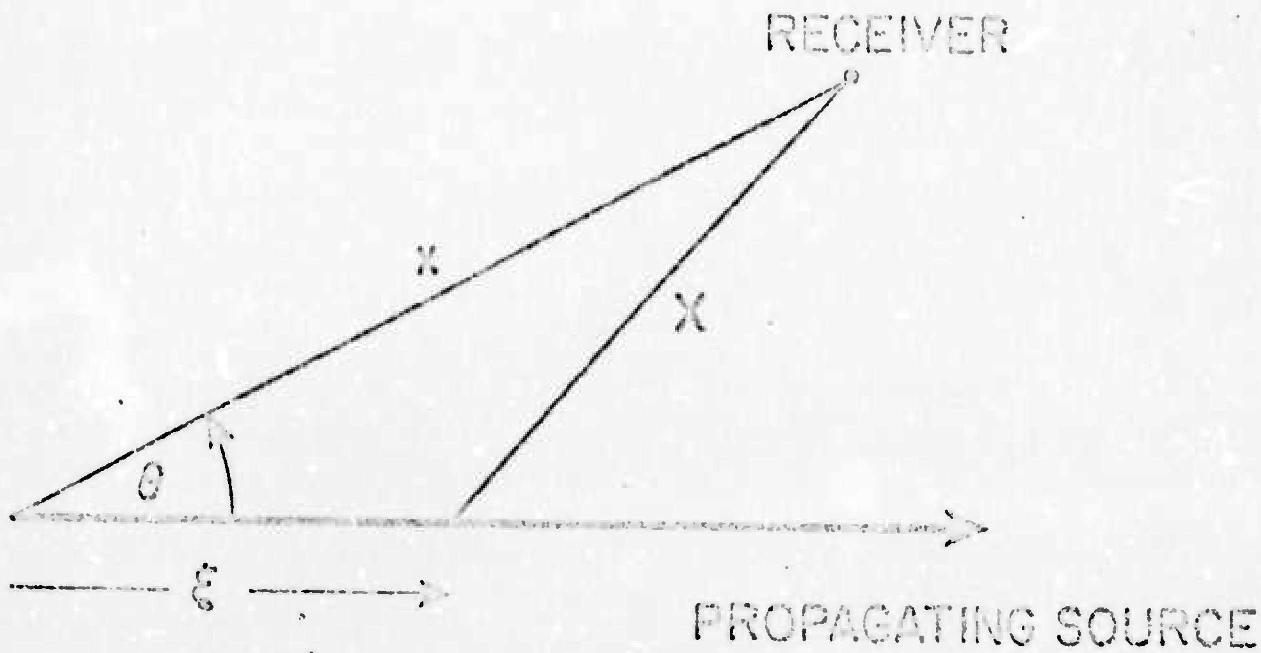

$V = V(\xi)$ is rupture velocity at position ξ.

$X = (x^2 + \xi^2 - 2x\xi \cos\theta)^{\frac{1}{2}}$.

Figure 1: Parameters for a propagating point source, generating surface waves at the receiver position.

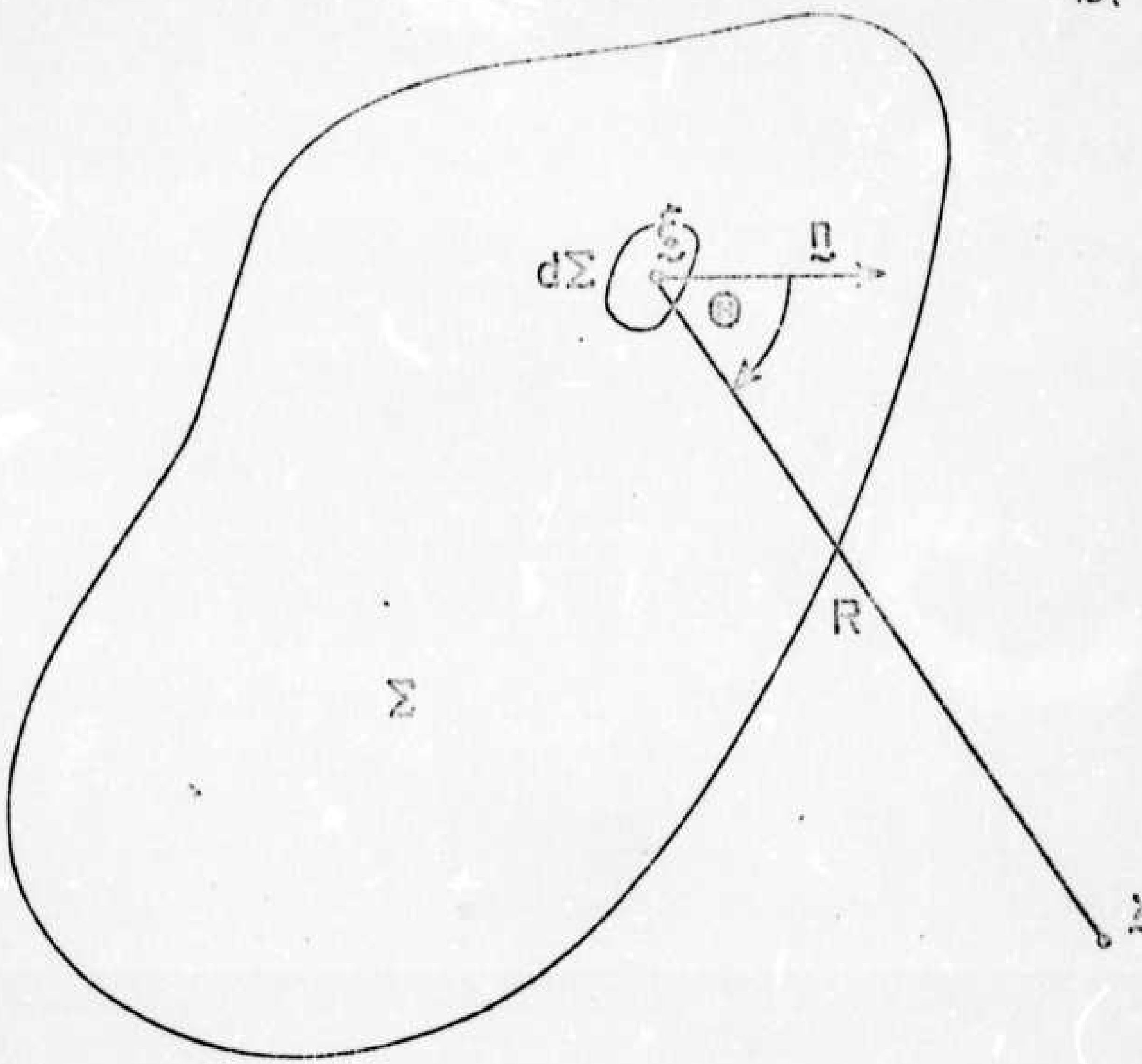

Figure 2: Parameters for a fault surface Σ, with area element $d\Sigma$ at $\underset{\sim}{\xi}$.